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Special Pythagorean Triangles And 9-Digit Dhuruva Numbers

M.A.Gopalan[1], K.Geetha[2], Manju Somanath[3]
[1]Professor, Department of Mathematics, Shrimathi Indira Gandhi College, Trichy,
[2]Assiatant Professor, Department of Mathematics, Cauvery College for Women, Trichy,
[3]Assiatant Professor, Department of Mathematics, National College, Trichy.

Abstract
Pythagorean triangles, each with a leg represented by a 9-digit Dhuruva number are obtained. A few interesting results are given.
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I. Introduction:

The fascinating branch of mathematics is the theory of numbers where in Pythagorean triangles have been a matter of interest to various mathematicians and to the lovers of mathematics, because it is a treasure house in which the search for many hidden connection is a treasure hunt. For a rich variety of fascinating problems one may refer [1-17].A careful observer of patterns may note that there is a one to one correspondence between the polygonal numbers and the number of sides of the polygon. Apart from the above patterns we have some more fascinating patterns of numbers namely Jarasandha numbers, Nasty numbers and Dhuruva numbers. These numbers have been presented in [18-21].

In [22-24], special Pythagorean triangles connected with polygonal numbers and Nasty numbers are obtained. Recently in [25], special Pythagorean triangles in connection with Hardy Ramanujan number 1729 and in [26], special Pythagorean triangles connection with 5 digit duruva number are exhibited. Thus the main objective of this paper is to find out the special Pythagorean triangles in connection with 9-digit Dhuruva numbers.

In this communication, we have presented Pythagorean triangles with each leg represented by 9-Digit Dhuruva numbers 554999445, 864197532 respectively. Also a few interesting results are obtained.

II. BASIC DEFINITONS:

Definition 2.1:

The ternary quadratic Diophantine equation given by $x^2+y^2=z^2$ is known as Pythagorean equation where x, y, z are natural numbers. The above equation is also referred to as Pythagorean triangle and denote it by T(x,y,z).

Also, in Pythagorean triangle T(x,y,z) : $x^2+y^2=z^2$, x and y are called its legs and z its hypotenuse.

Definition 2.2:

Most cited solution of the Pythagorean equation is $x=m^2-n^2, y=2mn, z=m^2+n^2$, where $m>n>0$. This solution is called primitive, if m,n are of opposite parity and gcd(m,n)=1.

Definition 2.3: Dhuruva numbers

The numbers which do not change when we perform a single operation or a sequence of operations are known as Dhuruva numbers.

III. METHOD OF ANALYSIS:

Section A:

In this section, we exhibit Pythagorean triangles, each with a leg represented by the nine digit Dhuruva number 554999445 and denote this number by N.

To start with, it is noted that the leg y can not be represented by N as y is even and N is odd. Also z can not be written as sum of two squares. Since a positive integer P can be written as a sum of two integer squares iff the canonical prime factorization $P=p_1^{r_1}p_2^{r_2}.....p_r^{r_r}$, (where p_i are distinct primes) satisfies the condition if $p_i\equiv 3(\mathrm{mod}\,4)$ then r_i is even. A prime $p\equiv 1(\mathrm{mod}\,4)$ can be written as $p=a^2+b^2$

Now, consider x=N

$\Rightarrow m^2-n^2=554999445$

which is a binary quadratic Diophantine equation. Solving the above equation for m,n, we get 49 integer solutions and thus, we have 49 pythagorean triangles, each having the leg x and the expression

$\frac{4A}{P} - y + z$ to be represented by the nine digit Dhuruva number N=554999445 as shown in the table below:

S.No	m	n	X, $\frac{4A}{P} - y + z$	y	z
1	18499989	18499974	554999445	684498631000572	684498631000797
2	55499947	55499942	554999445	6160487679006150	6160487679006170
3	7928581	7928546	554999445	125724238346452	125724238347677
4	6166683	6166638	554999445	76055403443508	76055403445533
5	5045477	5045422	554999445	50913121312588	50913121315613
6	4269259	4269194	554999445	36452589814492	36452589818717
7	2642907	2642802	554999445	13969359810828	13969359821853
8	2055621	2055486	554999445	8450600373612	8450600391837
9	1681899	1681734	554999445	5657013465732	5657013492957
10	1423173	1422978	554999445	4050287738388	4050287776413
11	881109	880794	554999445	1552151041092	1552151140317
12	720971	720586	554999445	1039043218012	1039043366237
13	685387	684982	554999445	938955516068	938955680093
14	610117	609662	554999445	743930300908	743930507933
15	560853	560358	554999445	628556930748	628557175773
16	500277	499722	554999445	499998845988	499999154013
17	474651	474066	554999445	450031801932	450032144157
18	388469	387754	554999445	301260817252	301261328477
19	240837	239682	554999445	115448587668	115449921693
20	203979	202614	554999445	82658002212	82659865437
21	214933	213638	554999445	91835712508	91837389533
22	187611	186126	554999445	69838569972	69840775197
23	167499	165834	554999445	55554058332	55556830557
24	158997	157242	554999445	50002012548	50005092573
25	137381	135346	554999445	37187937652	37192078877
26	81819	78354	554999445	12821691852	12833698077
27	73371	69486	554999445	10196514612	10211607837
28	69813	65718	554999445	9175941468	9192710493
29	57947	52942	554999445	6135660148	6160710173
30	48507	43242	554999445	4113587628	4150858653
31	55339	50074	554999445	5542090172	5569810397
32	46341	39906	554999445	3698567892	3739977117
33	42069	34854	554999445	2932545852	2984602077
34	41333	33962	554999445	2807502692	2861834333
35	36741	28194	554999445	2071751508	2144802717
36	35307	26298	554999445	1857006972	1938169053
37	33749	24166	554999445	1631156668	1722990557
38	22422	12321	554999445	1458461412	1560491613
39	31893	21498	554999445	1371271428	1479327453
40	30747	19758	554999445	1214998452	1335756573
41	29637	17982	554999445	1065865068	1201704093
42	29749	18166	554999445	1080840668	1215006557
43	28731	16446	554999445	945020052	1095941277
44	28683	16362	554999445	938622492	1090429533
45	27861	14874	554999445	828809028	997471197
46	26011	11026	554999445	573594572	798144797
47	25989	10974	554999445	570406572	795856797
48	25957	10898	554999445	565758772	792532253
49	25419	9546	554999445	485299548	737251677

Note that there are 28 primitive and 21 non-primitive pythagorean triangles.

Also, it is observed that there are 49 pythagorean triangles, where in each, the expressions $\frac{y+x-z}{2}$, $\frac{2A}{P}$ is represented by 554999445 as shown in table below:

S.No	m	n	x	y	z	$\frac{y+x-z}{2}$, $\frac{2A}{P}$
1	36999978	15	1368998372000260	1109999340	1368998372000710	554999445
2	110999894	5	12320976468011200	1109998940	12320976468011300	554999445
3	15857162	35	251449586693019	1110001340	251449586695469	554999445
4	12333366	45	152111916887931	1110002940	152111916891981	554999445
5	10090954	55	101827352627091	1110004940	101827352633141	554999445
6	8538518	65	72906289632099	1110007340	72906289640549	554999445
7	5285814	105	27939829631571	1110020940	27939829653621	554999445
8	4111242	135	16902310764339	1110035340	16902310800789	554999445
9	3363798	165	11315136957579	1110053340	11315137012029	554999445
10	2846346	195	8101685513691	1110074940	8101685589741	554999445
11	1762218	315	3105412180299	1110197340	3105412378749	554999445
12	1441942	385	2079196583139	1110295340	2079196879589	554999445
13	1370774	405	1879021195051	1110326940	1879021523101	554999445
14	1220234	455	1488970807731	1110412940	1488971221781	554999445
15	1121706	495	1258224105411	1110488940	1258224595461	554999445
16	1000554	555	1001107998891	1110614940	1001108614941	554999445
17	949302	585	901173944979	1110683340	901174629429	554999445
18	776938	715	603632144619	1111021340	603633167069	554999445
19	481674	1155	232008508251	1112666940	232011176301	554999445
20	429866	1295	184783100931	1113352940	184786454981	554999445
21	407958	1365	166427866539	1113725340	166431592989	554999445
22	375222	1485	140789344059	1114409340	140793754509	554999445
23	334998	1665	112220887779	1115543340	112226432229	554999445
24	317994	1755	101117104011	1116158940	101123264061	554999445
25	274762	2035	75490015419	1118281340	75498297869	554999445
26	163638	3465	26765388819	1134011340	26789401269	554999445
27	146742	3885	21518121339	1140185340	21548307789	554999445
28	139626	4095	19478650851	1143536940	19512188901	554999445
29	115894	5005	13406369211	1160098940	13456469261	554999445
30	110678	5265	12221899459	1165439340	12277339909	554999445
31	97014	6105	9374445171	1184540940	9448987221	554999445
32	92682	6435	8548543899	1192817340	8631362349	554999445
33	84138	7215	7027146819	1214111340	7131259269	554999445
34	82666	3371	6822303915	557334172	6845031197	554999445
35	73482	8547	5326553115	1256101308	5472655533	554999445
36	70614	9009	4905174915	1272323052	5067499077	554999445
37	67498	9583	4464146115	1293666668	4647813893	554999445
38	65046	10101	4128951915	1314059292	4333012317	554999445
39	63786	10395	3960597771	1326110940	4176709821	554999445
40	61494	10989	3660753915	1351515132	3902270157	554999445
41	59274	11655	3377568051	1381676940	3649246101	554999445
42	59498	11583	3405846115	1378330668	3674177893	554999445
43	57462	12285	3150960219	1411841340	3452802669	554999445
44	57366	12321	3139050915	1413612972	3442664997	554999445
45	55722	12987	2936279115	1447323228	3273603453	554999445
46	52022	14985	2481738259	1559099340	2930838709	554999445
47	51978	15015	2476262259	1560899340	2927162709	554999445
48	51914	15059	2468289915	1563545852	2921836877	554999445
49	50838	15873	2332550115	1613903148	2836454373	554999445

Note that there are 18 primitive and 31 non-primitive pythagorean triangles.

Also, it is noted that the expressions $\frac{z+x-y}{2}$, $x-\frac{2A}{P}$ are represented respectively by the dhuruva number 554999445 as shown in table below:

S.No	m	n	x	y	z	$\frac{z+x-y}{2}$, $x-\frac{2A}{P}$
1	1370369	1369964	1109834865	3754712393432	3754712557457	554999445
2	1219779	1219324	1109791865	2974611618792	2974611825817	554999445
3	36999963	36999948	1109998665	2737993414003850	2737993414004070	554999445
4	110999889	110999884	1109998864	24641949606025800	24641949606025800	554999445
5	15857127	15857092	1109997665	502895843389368	502895843390593	554999445
6	12333321	12333276	1109996865	304220503779192	304220503781217	554999445
7	10090899	10090844	1109995865	203651375257512	203651375260537	554999445
8	8538453	8538388	1109994665	145809249267528	145809249271753	554999445
9	5285709	5285604	1109987865	55876329266472	55876329277497	554999445
10	4111107	4110972	1109980665	33801291532008	33801291550233	554999445
11	3363633	3363468	1109971665	22626943918488	22626943945713	554999445
12	2846151	2845956	1109960865	16200041030712	16200041068737	554999445
13	1761903	1761588	1109899665	6207494363928	6207494463153	554999445
14	1441557	1441172	1109850665	4155063169608	4155063317833	554999445
15	1121211	1120716	1109753865	2513118214152	2513118459177	554999445
16	999999	999444	1109690865	1998886001112	1998886309137	554999445
17	948717	948132	1109656665	1799017893288	1799018235513	554999445
18	776223	775508	1109487665	1203934292568	1203934803793	554999445
19	480519	479364	1108664865	460687019832	460688353857	554999445
20	428571	427276	1108321865	366236205192	366237882217	554999445
21	406593	405228	1108135665	329525736408	329527599633	554999445
22	373737	372252	1107793665	278248691448	278250896673	554999445
23	333333	331668	1107226665	221111778888	221114551113	554999445
24	316239	314484	1106918865	198904211352	198907291377	554999445
25	272727	270692	1105857665	147650034168	147654175393	554999445
26	160173	156708	1097992665	50200780968	50212787193	554999445
27	142857	138972	1094905665	39706246008	39721339233	554999445
28	135531	131436	1093229865	35627305032	35644074057	554999445
29	110889	105884	1084948865	23482741752	23507791777	554999445
30	105413	100148	1082278665	21113802248	21141522473	554999445
31	90909	84804	1072727865	15418893672	15456164697	554999445
32	86247	79812	1068589665	13767091128	13808500353	554999445
33	76923	69708	1057942665	10724296968	10776353193	554999445
34	75295	67924	1055667249	10228675160	10283006801	554999445
35	64935	56388	1036947681	7323109560	7396160769	554999445
36	61605	52596	1028836809	6480353160	6561515241	554999445
37	57915	48332	1018165001	5598295560	5690129449	554999445
38	54945	44844	1007968689	4927907160	5029937361	554999445
39	53391	42996	1001942865	4591198872	4699254897	554999445
40	50505	39516	989240769	3991511160	4112269281	554999445
41	47619	35964	974159865	3425139432	3560978457	554999445
42	47915	36332	975833001	3481695560	3615861449	554999445
43	45177	32892	959077665	2971923768	3122844993	554999445
44	45045	32724	958191849	2948105160	3099912201	554999445
45	42735	29748	941336721	2542561560	2711223729	554999445
46	37037	22052	885448665	1633479848	1858030073	554999445
47	36963	21948	884548665	1622527848	1847978073	554999445
48	36855	21796	883225409	1606583160	1833356641	554999445
49	34965	19092	858046761	1335103560	1587055689	554999445

Note that there are 20 primitive and 29 non-primitive pythagorean triangles.

Section B:

On following the procedure of section A, we get 15 integer solutions and thus, we have 15 pythagorean triangles, each having the leg y to be represented by the nine digit Dhuruva number N=864197532 as shown in the table below:

S.No	m	n	x	y	z
1	216049383	2	46677335894680700	864197532	46677335894680700
2	144032922	3	20745482619858100	864197532	20745482619858100
3	72016461	6	5186370654964480	864197532	5186370654964560
4	48010974	9	2305053624428590	864197532	2305053624428760
5	39281709	11	1543052661960560	864197598	1543052661960800
6	24005487	18	576263406106845	864197532	576263406107493
7	19640853	22	385763106567125	864197532	385763106568093
8	16003658	27	256117069380235	864197532	256117069381693
9	13093902	33	171450269584515	864197532	171450269586693
10	8001829	54	64029267342325	864197532	64029267348157
11	6546951	66	42862567392045	864197532	42862567400757
12	4364634	99	19050029944155	864197532	19050029963757
13	2387286	181	5699134413035	864197532	5699134478557
14	2182317	198	4762507449285	864197532	4762507527693
15	727439	594	529167145885	864197532	529167851557

Note that there are 8 primitive and 7 non-primitive pythagorean triangles.

Also, it is noted that the expressions $\frac{z+x-y}{2}$, $x-\frac{2A}{P}$ are represented respectively by the dhuruva number 864197532 as shown in table below:

S. No	m	n	x	y	z	$\frac{z+x-y}{2}$, $x-\frac{2A}{P}$
1	432098766	432098764	1728395072	373418685429050000	373418685429050000	864197532
2	288065844	288065841	1728395056	165963859230470000	165963859230470000	864197532
3	216049383	216049379	1728395048	93354670060966300	93354670060966300	864197532
4	144032922	144032916	1728395028	41490963511321100	41490963511321100	864197532
5	96021948	96021939	1728394984	18440427267034300	18440427267034400	864197598
6	78563412	78563401	1728394943	12344417681768400	12344417681768500	864197532
7	72016461	72016449	1728394920	10372739581534000	10372739581534100	864197532
8	48010974	48010956	1728394740	4610105520462290	4610105520462610	864197532
9	39281706	39281684	1728394580	3086103124145810	3086103124146290	864197532
10	32007316	32007289	1728394335	2048934826652650	2048934826653380	864197532
11	26187804	26187771	1728393975	1371600428289770	1371600428290860	864197532
12	24005487	24005451	1728393768	1152525083819270	1152525083820570	864197532
13	19640853	19640809	1728393128	771524484740154	771524484742090	864197532
14	16003658	16003604	1728392148	512232410366864	512232410369780	864197532
15	13093902	13093836	1728390708	342898810776144	342898810780500	864197532
16	8729268	8729169	1728385263	152398511236584	152398511246385	864197532
17	8001829	8001721	1728383400	128056806295418	128056806307082	864197532
18	6546951	6546819	1728377640	85723406397738	85723406415162	864197532
19	4774572	4774391	1728362303	45591347171304	45591347204065	864197532
20	4364634	4364436	1728355860	38098331512848	38098331552052	864197532
21	727439	726251	1726983720	1056606602378	1056608013722	864197532
22	2182317	2181921	1728238248	9523286581914	9523286738730	864197532

Note that there are 6 primitive and 16 non-primitive pythagorean triangles.

Also it is noted that leg x to be represented by the nine digit Dhuruva number N=864197532 as shown in the table below:

S.No.	m	n	x	y	z

1	72016464	72016458	864197532	10372741309929000	10372741309929100
2	24005496	24005478	864197532	1152526812214180	1152526812214500
3	8001856	8001802	864197532	128058534689024	128058534691940

The above 3 are non-primitive pythagorean triangles.

Also, it is noted that the expressions $\frac{y+x-z}{2}$, $\frac{2A}{P}$ are represented respectively by the dhuruva number 864197532 as shown in table below:

S. No.	m	n	x	y	z	$\frac{y+x-z}{2}$, $\frac{2A}{P}$
1	432098768	432098766	1728395072	373418688885841000	373418688885841000	864197532
2	288065844	288065841	1728395056	165963859230470000	165963859230470000	864197532
3	216049383	216049379	1728395048	93354670060966300	93354670060966300	864197532
4	144032928	144032922	1728395100	41490966968111200	41490966968111300	864197532
5	96021957	96021948	1728395144	18440430723824500	18440430723824600	864197598
6	78563423	78563412	1728395185	12344421138558600	12344421138558700	864197532
7	72016461	72016449	1728394920	10372739581534000	10372739581534100	864197532
8	48010992	48010974	1728395388	4610108977252420	4610108977252740	864197532
9	39281728	39281706	1728395548	3086106580935940	3086106580936420	864197532
10	32007343	32007316	1728395793	2048938283442780	2048938283443500	864197532
11	26187837	26187804	1728396153	1371603885079900	1371603885080980	864197532
12	24005523	24005487	1728396360	1152528540609400	1152528540610700	864197532
13	19640897	19640853	1728397000	771527941530282	771527941532218	864197532
14	16003712	16003658	1728397980	512235867156992	512235867159908	864197532
15	13093968	13093902	1728399420	342902267566272	342902267570628	864197532
16	8729367	8729268	1728404865	152401968026712	152401968036513	864197532
17	8001829	8001721	1728383400	128056806295418	128056806307082	864197532
18	6546951	6546819	1728377640	85723406397738	85723406415162	864197532
19	4774572	4774391	1728362303	45591347171304	45591347204065	864197532
20	4364634	4364436	1728355860	38098331512848	38098331552052	864197532
21	727439	726251	1726983720	1056606602378	1056608013722	864197532
22	2182317	432098766	1728238248	9523286581914	9523286738730	864197532

Note that there are 7 primitive and 15 non-primitive pythagorean triangles.

IV. CONCLUSION

One may search for the connections between Pythagorean triangles and other Dhuruva numbers.

REFERENCES:

[1]. W. Sierpinski, Pythagorean triangles, Dover publications, INC, Newyork, 2003.

[2]. M. A. Gopalan and G.Janaki, "*Pythagorean triangle with area/perimeter as a special polygonal number*", Bulletin of Pure and Applied Science, Vol.27E (No.2), 393-402, 2008.

[3]. M. A. Gopalan and A. Vijayasankar, "*Observations on a Pythagorean problem*", Acta Ciencia Indica, Vol. XXXVI M, No 4, 517-520, 2010.

[4]. M. A. Gopalan and S. Leelavathi, "*Pythagorean triangle with area/perimeter as a square integer*", International Journal of Mathematics, Computer sciences and information Technology, Vol.1, No.2, 199-204, 2008.

[5]. M.A.Gopalan and A.Gnanam, "*Pairs of Pythagorean triangles with equal perimeters*", Impact J.Sci.Tech., Vol 1(2), 67-70, 2007.

[6]. M.A.Gopalan and S.Leelavathi, "*Pythagorean triangle with 2 area/perimeter as a cubic integer*", Bulletin of Pure and Applied Science, Vol.26E (No.2), 197-200,2007.

[7]. M.A.Gopalan and A.Gnanam, "*A special Pythagorean problem*", Acta Ciencia Indica, Vol. XXXIII M, No 4, 1435-1439,2007.

[8]. M. A. Gopalan, A. Gnanam and G.Janaki, "*A Remarkable Pythagorean problem*", Acta Ciencia Indica, Vol. XXXIII M, No 4, 1429-1434,2007.

[9]. M.A.Gopalan, and S. Devibala, "*On a Pythagorean problem*", Acta Ciencia Indica, Vol. XXXII M, No 4, 1451-1452, 2006.

[10]. M.A.Gopalan and B.Sivakami, "*Special Pythagorean triangles generated through the integral solutions of the equation*

$y^2 = (k^2 + 2k)x^2 + 1$ ", Diophantus J. Math., Vol 2(1), 25-30, 2013.

[11]. M. A. Gopalan and A. Gnanam, "*Pythagorean triangles and Polygonal numbers*", International Journal of Mathematical Sciences, Vol 9, No. 1-2, 211-215,2010.

[12]. K. Meena, S. Vidhyalakshmi, B. Geetha, A. Vijayasankar and M. A. Gopalan," *Relations between special polygonal numbers generated through the solutions of Pythagorean equation*",IJISM, Vol. 2(2),257-258,2014.

[13]. M.A.Gopalan and G.Janaki, "*Pythagorean triangle with perimeter as Pentagonal number*", Antarctica J.Math., Vol 5(2), 15-18, 2008.

[14]. M. A. Gopalan and G. Sangeetha, "*Pythagorean triangle with perimeter as triangular number*",GJ-AMMS,Vol. 3, No 1-2,93-97,2010.

[15]. M.A.Gopalan, Manjusomanath and K. Geetha," *Pythagorean triangle with area/perimeter as a Special polygonal number*", IOSR-JM, Vol. 7(3),52-62,2013.

[16]. M.A. Gopalan and V. Geetha," *Pythagorean triangle with area/perimeter as a Special polygonal number*", IRJES, Vol.2(7),28-34,2013.

[17]. M. A. Gopalan and B. Sivakami, "*Pythagorean triangle with hypotenuse minus 2(area/ perimeter) as a square integer*", Archimedes J.Math., Vol 2(2), 153-166, 2012.

[18]. J.N.Kapur, Dhuruva numbers, Fasinating world of Mathematics and Mathematical sciences, Trust society, Vol 17, 1997.

[19]. Bert Miller, Nasty numbers, The mathematics teacher, No.9,Vol 73,649,1980.

[20]. Charles Bown. K, Nasties are primitives, The mathematics teacher, No.9, Vol 74,502-504, 1981.

[21]. P.S.N.Sastry, Jarasandha numbers, The mathematics teacher, No.9,Vol 37,issues 3 & 4 , 2001.

[22]. M. A. Gopalan V. Sangeetha and Manjusomanath, "*Pythagorean triangle and Polygonal number*", Cayley J.Math., Vol 2(2),151-156, 2013.

[23]. M.A.Gopalan and G.Janaki, " *Pythagorean triangle with nasty number as a leg*",Journal of applied Mathematical Analysis and Applications, Vol 4,No 1-2,13-17,2008.

[24]. M.A.Gopalan and S.Devibala, "*Pythagorean triangle with Triangular number as a leg*", Impact J.Sci.Tech., Vol 2(4), 195-199, 2008.

[25]. Dr.Mita Darbari, *A connection between Hardy-Ramanujan number and special Pythagorean triangle*," Bulletin of society for Mathematical services & Standards, Vol 3, No.2, 71-73, 2014.

[26]. M. A. Gopalan, S. Vidhyalakshmi, E. Premalatha and K. Presenna, "*Special Pythagorean triangles and 5-digit dhuruva number*", International Research Journal of Mathematics, Engineering and IT, Vol.1, Issue.4, 29-33, Aug 2014.